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Inflow-outflow problems for Euler equations in a rectangular cylinder

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Abstract. We prove that some inflow-outflow problems for the Euler equations in a (nonsmooth) bounded cylinder admit a regular solution. The problems considered are symmetric hyperbolic systems with partly characteristic and partly noncharacteristic boundary; for such problems, no general theory is available. Therefore, we introduce particular spaces of functions satisfying suitable additional boundary conditions which allow to determine a regular solution by means of a "reflection technique".

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1 Introduction

Let Ω be an open bounded cylinder in $\mathbb{R}^3$ having a rectangular section: more precisely, let Ω be the cartesian product of an open rectangle R with a bounded interval (a, b) ($\Omega = R \times (a, b)$) and let $\Gamma_1 = \overline{R} \times \{a, b\}$ and $\Gamma_0 = \partial R \times [a, b]$ so that the piecewise smooth boundary $\partial\Omega$ may be characterized by $\partial\Omega = \Gamma_1 \bigcup \Gamma_0$. In the cylinder Ω we consider the following initial-boundary value problem for the Euler equations for a barotropic inviscid compressible fluid

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho v) = 0 & \text{in } [0,T] \times \Omega \\ \rho(\partial_t v + (v \cdot \nabla) v - f) + \nabla p = 0 & \text{in } [0,T] \times \Omega \\ M(\rho, v) = G & \text{on } (0,T) \times \Gamma_1 \\ v \cdot \nu = 0 & \text{on } (0,T) \times \Gamma_0 \\ \rho(0,x) = \rho_0(x) & \text{in } \Omega \\ v(0,x) = v_0(x) & \text{in } \Omega, \end{cases} \tag{1}$$

where $\partial_t = \partial/\partial t$, M is a matrix which depends on the particular inflow-outflow problem considered, see (6), (9), and ν denotes the unit outward normal to $\partial\Omega$, when it exists; the density $\rho = \rho(x,t)$, the velocity field $v = v(t,x) = (v_1, v_2, v_3)$ and the pressure $p = p(t,x)$ are unknown functions of time $t \in (0,T)$ and space variable $x \in \Omega$. In (1) the density ρ is assumed to be positive for physical reasons; moreover, ρ and p are related by the equation of state $p = p(\rho)$ where $p : \mathbb{R}_+ \to \mathbb{R}_+$ is a given smooth function such that $p'(s) > 0$ for all $s > 0$. The external force field $f = f(t,x)$, the boundary data G and the initial data v_0 and ρ_0 are known functions.

Usually, the Euler system is studied under the slip boundary condition $v \cdot \nu = 0$ on Γ_0, see [1, 2, 3, 9]. In this paper we study the existence and uniqueness of a regular solution for some inflow-outflow problems for (1): we assume that the fluid flows in on $\Gamma_1^i = \overline{R} \times \{a\}$, that it flows out on $\Gamma_1^o = \overline{R} \times \{b\}$ and satisfies the slip condition $v \cdot \nu = 0$ on Γ_0. On $\Gamma_1 = \Gamma_1^i \cup \Gamma_1^o$ the flow is assumed to be either supersonic or subsonic. As far as we are aware, inflow-outflow problems for (1) have only been studied in [12] provided that there is no "discontinuity" in the boundary conditions. The existence of a regular solution of (1) is not at all obvious: first because the domain Ω is nonsmooth, second because the boundary matrix does not have constant rank. Nevertheless, if suitable functional spaces are considered, the presence of a nonsmooth boundary avoids in some sense the problems arising from boundary points where the boundary matrix changes rank and enables us to prove the existence of a regular solution. In next section we write (1) as a symmetric hyperbolic system [4]; with our assumptions it becomes partly characteristic and partly noncharacteristic: more precisely, the boundary Γ_1 is noncharacteristic while the boundary Γ_0 is characteristic of rank two. To our knowledge no general theory is available for these problems even if some partial results may be found in [6, 7, 8, 11, 13].

Our method consists in introducing a class of functional spaces of functions having some vanishing traces; these spaces allow, by a "reflection technique", to reduce the linearized problem associated to (1) to a noncharacteristic problem. Then, standard existence results apply and we obtain a solution of the linearized problem in Ω by a partition of unity. Finally, the solution of the nonlinear problem (1) is obtained by a fixed point argument. With this method, we prove the existence and uniqueness of a regular local in time solution (ρ, v) of (1) for both the supersonic and the subsonic cases: we assume that the fluid flows in Γ_1^i either supersonic or subsonic and analogously on the outflow part Γ_1^o. Even if the underlying abstract framework is similar, these two cases turn out to be quite different: the subsonic case is slightly more delicate because nonlinear boundary conditions have to be considered.

2 Notations and results

For simplicity, we take

$$\Omega = (0,2) \times (0,1) \times (0,1), \tag{2}$$

we denote $Q_T = (0,T) \times \Omega$, $\Sigma_T = [0,T] \times \partial\Omega$ and we define the sets

$$\Gamma_1^i = \{0\} \times [0,1] \times [0,1] \qquad \Gamma_1^o = \{2\} \times [0,1] \times [0,1] \qquad \Gamma_1 = \Gamma_1^i \bigcup \Gamma_1^o$$

$$\Gamma_2 = [0,2] \times \{0,1\} \times [0,1] \qquad \Gamma_3 = [0,2] \times [0,1] \times \{0,1\} \qquad \Gamma_0 = \Gamma_2 \bigcup \Gamma_3;$$

then, the piecewise smooth boundary $\partial\Omega$ is given by $\partial\Omega = \Gamma_1 \bigcup \Gamma_2 \bigcup \Gamma_3$.

Let $H^m(\Omega)$ be the usual Sobolev space of order m ($m \in \mathbb{N}$) and let $\|\cdot\|_m$ denote its norm: the $L^2(\Omega)$-norm is simply denoted by $\|\cdot\|$. We also introduce the following spaces of functions having some (even or odd) traces vanishing on parts of the boundary

$$\mathcal{H}_e^3 = \{\phi \in H^3(\Omega);\ \phi = 0 \text{ on } \Gamma_2,\ \partial_3\phi = 0 \text{ on } \Gamma_3,\ \partial_2^2\phi = 0 \text{ on } \Gamma_2\}$$

$$\mathcal{H}_o^3 = \{\phi \in H^3(\Omega);\ \phi = 0 \text{ on } \Gamma_3,\ \partial_2\phi = 0 \text{ on } \Gamma_2,\ \partial_3^2\phi = 0 \text{ on } \Gamma_3\}$$

$$\mathcal{H}^3 = \{\phi \in H^3(\Omega);\ \partial_\nu\phi = 0 \text{ on } \Gamma_0\};$$

clearly, these are closed subspaces of $H^3(\Omega)$ and contain $H_0^3(\Omega)$: here, $\partial_\nu = \partial/\partial\nu$, $\partial_i = \partial/\partial x_i$, $\partial_i^2 = \partial^2/\partial x_i^2$ and the traces are well-defined even if the domain Ω is nonsmooth, see [5]. Similarly, we define the spaces $\mathcal{H}^3(\Gamma_1)$, $\mathcal{H}_o^3(\Gamma_1)$ and $\mathcal{H}_e^3(\Gamma_1)$. For any vector function ϕ defined on (a subset of) $\bar{Q}_T$ we denote by ϕ_i its i-th component, $i = 1,2,3$. Let B be a Banach space and let $T > 0$: then $C(0,T;B)$ and $L^\infty(0,T;B)$ denote respectively the space of continuous and essentially bounded functions defined on $[0,T]$ and taking values in B. Define the space

$$\mathcal{C}_T(\mathcal{H}_*^3) = C(0,T;\mathcal{H}^3) \times C(0,T;\mathcal{H}^3) \times C(0,T;\mathcal{H}_e^3) \times C(0,T;\mathcal{H}_o^3).$$

Consider now the spaces

$$\mathcal{C}_T(H^3) \ = \ \bigcap_{k=0}^{3} \ C^k(0,T;\ H^{3-k}(\Omega))$$

$$\mathcal{L}_T^\infty(H^3) \ = \ \bigcap_{k=0}^{3} \ W^{k,\infty}(0,T;\ H^{3-k}(\Omega))$$

with the norm given by ($\partial_t^k = \partial^k/\partial t^k$):

$$|||u|||_{3,T} = \sup_{[0,T]} |||u(t)|||_3, \qquad |||u(t)|||_3^2 = \sum_{k=0}^{3} \|\partial_t^k u(t)\|_{3-k}^2.$$

We seek solutions (ρ, v) of (1) in the closed subspace of $[\mathcal{C}_T(H^3)]^4$ defined by

$$\mathcal{K}_T = \mathcal{C}_T(\mathcal{H}_*^3) \cap [\mathcal{C}_T(H^3)]^4.$$

Finally, consider the space

$$H_*^3(Q_T) = \{\phi \in [H^3(Q_T)]^3;\ \phi(t) \in \mathcal{H}^3 \times \mathcal{H}_e^3 \times \mathcal{H}_o^3 \text{ for a.e. } t \in [0,T]\}$$

normed by

$$[\phi]_{3,T}^2 = \int_0^T |||\phi(t)|||_3^2 \ dt;$$

similarly, we define $H_*^3(\Sigma_T)$ and the norm $[\cdot]_{3,\Sigma_T}$.

By introducing the sound velocity $c(\rho) = \sqrt{p'(\rho)}$, the equations in problem (1) become

$$\begin{cases} \frac{1}{\rho}(\partial_t \rho + v \cdot \nabla \rho) + \nabla \cdot v = 0 & \text{in } [0,T] \times \Omega \\ \frac{\rho}{c^2(\rho)}(\partial_t v + (v \cdot \nabla) v - f) + \nabla \rho = 0 & \text{in } [0,T] \times \Omega, \end{cases} \tag{3}$$

to which we associate the initial conditions

$$\begin{cases} \rho(0,x) = \rho_0(x) & \text{in } \Omega \\ v(0,x) = v_0(x) & \text{in } \Omega, \end{cases} \tag{4}$$

for ρ_0 and v_0 in a suitable functional space.

We may write (3) as a quasilinear symmetric hyperbolic system, that is in the form

$$A_0(u)\partial_t u + \sum_{j=1}^{3} A_j(u)\partial_j u = F(u) \tag{5}$$

where $u = (\rho, v)$, the A_j $(j = 0, \ldots, 3)$ are symmetric and A_0 is positive definite. The boundary matrix is

$$A_\nu = \begin{pmatrix} \frac{1}{\rho} v \cdot \nu & \nu^T \\ \nu & \frac{\rho}{c^2(\rho)} v \cdot \nu I_3 \end{pmatrix};$$

therefore, the matrix $A_0^{-1} A_\nu$ has eigenvalues

$$v \cdot \nu, \quad v \cdot \nu, \quad v \cdot \nu + c(\rho), \quad v \cdot \nu - c(\rho).$$

Recall that a part Γ of the boundary is said noncharacteristic if $\det A_\nu \neq 0$ on Γ; moreover, if we are given homogeneous boundary conditions $Mu = 0$, then $\ker M$ is said to be maximally non-negative for A_ν if

$$(A_\nu u, u) \geq 0 \qquad \text{on } \partial\Omega \times (0,T) \qquad \forall u \in \ker M,$$

and $\ker M$ is not properly contained in any other subspace having this property; the maximal non-negativity is useful because it allows to neglect some boundary integrals when one looks for a priori estimates. This condition corresponds to requiring that the number of boundary conditions equals the number of negative eigenvalues of A_ν. Let us first consider the supersonic case: on the inflow part of the boundary Γ_1^i (which is noncharacteristic) the four eigenvalues of $A_0^{-1}A_\nu$ are negative and the whole state of the fluid must be prescribed while on the outflow part Γ_1^o the boundary matrix $A_0^{-1}A_\nu$ is positive definite and no condition has to be assigned. For the subsonic case we also have that Γ_1 is noncharacteristic: on Γ_1^i three eigenvalues of $A_0^{-1}A_\nu$ are negative and three conditions must be prescribed while on Γ_1^o the boundary matrix $A_0^{-1}A_\nu$ only has a negative eigenvalue and one condition has to be assigned. Finally, the impermeable part Γ_0 is characteristic of rank two with $A_0^{-1}A_\nu$ having one negative eigenvalue and only the condition $v \cdot \nu = 0$ is given.

So, if we fix $T_0 > 0$ and we take into account the shape of Ω in (2), for the supersonic case we require that

$$\begin{cases} (\rho, v) = (r, g) & \text{on } (0,T_0) \times \Gamma_1^i \\ v_2 = 0 & \text{on } (0,T_0) \times \Gamma_2 \\ v_3 = 0 & \text{on } (0,T_0) \times \Gamma_3 \end{cases} \tag{6}$$

while no conditions should be imposed on Γ_1^o; however, to ensure that the fluid flows out supersonic at least in a small interval of time, we require that the initial flow satisfies such condition: we assume that

$$\begin{aligned} &\rho_0 > 0 && \text{in } \bar{\Omega} \\ &(v_0)_1 > c(\rho_0) && \text{on } \Gamma_1 \\ &r > 0 && \text{on } [0,T_0] \times \Gamma_1^i \\ &g_1 > c(r) && \text{on } [0,T_0] \times \Gamma_1^i. \end{aligned} \tag{7}$$

Finally, to find regular solutions one needs to impose some necessary compatibility conditions between the boundary data and the initial values; denote by $\partial_t^k \rho_0$ and $\partial_t^k v_0$ the functions obtained by formally taking $k-1$ time derivatives of (3), solving for $\partial_t^k \rho$ and $\partial_t^k v$ and evaluating at time $t = 0$. Then the compatibility conditions in the supersonic case read

$$\partial_t^k \rho_0 = \partial_t^k r(0), \quad \partial_t^k v_0 = \partial_t^k g(0) \ \text{ on } \Gamma_1^i \quad \partial_t^k v_0 \cdot \nu = 0 \ \text{ on } \Gamma_0 \quad k = 0,1,2; \tag{8}$$

for $u_0 = (\rho_0, v_0)$ we set $|||u_0|||_m^2 = \sum_{k=1}^m (\|\partial_t^k \rho_0\|_{m-k}^2 + \|\partial_t^k v_0\|_{m-k}^2)$.

Our main result in the supersonic case states

Theorem 1 *Let $\Omega \subset \mathbb{R}^3$ be as in* (2)*, let $T_0 > 0$ and assume that:*

(i) *the boundary data $r, g \in H^3((0,T_0) \times \Gamma_1^i)$ satisfy*

$$(r,g) \in \mathcal{H}^3(\Gamma_1^i) \times \mathcal{H}^3(\Gamma_1^i) \times \mathcal{H}_e^3(\Gamma_1^i) \times \mathcal{H}_o^3(\Gamma_1^i)$$

$$\textit{for a.e. } t \in [0,T_0];$$

(ii) *the initial data satisfy $(\rho_0, v_0) \in \mathcal{H}^3 \times \mathcal{H}^3 \times \mathcal{H}_e^3 \times \mathcal{H}_o^3$;*

(iii) *the external force satisfies $f \in H_*^3(Q_{T_0})$;*

(iv) (7) *and* (8) *hold.*

Then, there exists $T > 0$ such that (3)–(4)–(6) *admits a unique solution $(\rho, v) \in \mathcal{K}_T$ satisfying $v_1 > c(\rho)$ on $[0,T] \times \Gamma_1^o$.*

Remark. Some of the compatibility conditions (8) are "hidden" in the assumptions (i)–(iii); further regularity may be obtained by using Sobolev spaces H^m of higher order ($m \geq 4$): in such case, one must obviously strengthen (8) with supplementary conditions.

In the subsonic case we assign the tangential velocity of the fluid on the inflow part and the normal velocity on the outflow part; we require that

$$\begin{cases} v_2 = g_2, \quad v_3 = g_3, \quad v_1 + c(\rho) = \psi & \text{on } (0,T_0) \times \Gamma_1^i \\ v_1 = g_1 & \text{on } (0,T_0) \times \Gamma_1^o \\ v_2 = 0 & \text{on } (0,T_0) \times \Gamma_2 \\ v_3 = 0 & \text{on } (0,T_0) \times \Gamma_3 \end{cases} \tag{9}$$

and we assume that

$$\begin{array}{ll} \rho_0 > 0 & \text{in } \bar{\Omega} \\ -c(\rho_0) < -\psi(0) + c(\rho_0) < 0 & \text{on } \Gamma_1^i \\ 0 < g_1(0) < c(\rho_0) & \text{on } \Gamma_1^o \\ g_1 > 0 & \text{on } [0,T_0] \times \Gamma_1^o. \end{array} \tag{10}$$

In order to ensure that the fluid flows in subsonic on Γ_1^i one should require that $-c(\rho) < -\psi + c(\rho) < 0$ on $[0,T_0] \times \Gamma_1^i$ while to ensure that the fluid flows out subsonic on Γ_1^o one should require that $g_1 < c(\rho)$ on $[0,T_0] \times \Gamma_1^o$ but these conditions depend on the solution; nevertheless, we will prove in Theorem 2 below that they hold in some interval of time $[0,T]$, $T > 0$. We also refer to [12] for the derivation of (9) and for other possible boundary conditions.

The compatibility conditions between the initial and boundary data in the subsonic case read

$$\begin{aligned}
&(\partial_t^k v_0)_2 = \partial_t^k g_2(0), \quad (\partial_t^k v_0)_3 = \partial_t^k g_3(0), \\
&(\partial_t^k v_0)_1 + \partial_t^k c_0 = \partial_t^k \psi(0) \quad \text{on } \Gamma_1^i \\
&(\partial_t^k v_0)_1 = \partial_t^k g_1(0) \quad \text{on } \Gamma_1^o \\
&\partial_t^k v_0 \cdot \nu = 0 \quad \text{on } \Gamma_0 \quad k = 0,1,2,
\end{aligned} \tag{11}$$

where $\partial_t^k c_0$ denotes the k^{th} time derivative at $t = 0$ of $c(\rho)$. Then, for the subsonic case we prove the following

Theorem 2 *Let $\Omega \subset \mathbb{R}^3$ be as in (2), let $T_0 > 0$ and assume that:*

(i) $\psi, g_2, g_3 \in H^3((0,T_0) \times \Gamma_1^i)$, $g_1 \in H^3((0,T_0) \times \Gamma_1^o)$ *satisfy*

$$g_2 \in \mathcal{H}_e^3(\Gamma_1^i), \quad g_3 \in \mathcal{H}_o^3(\Gamma_1^i), \quad \psi \in \mathcal{H}^3(\Gamma_1^i), \quad g_1 \in \mathcal{H}^3(\Gamma_1^o)$$

for a.e. $t \in [0, T_0]$;

(ii) *the initial data satisfy* $(\rho_0, v_0) \in \mathcal{H}^3 \times \mathcal{H}^3 \times \mathcal{H}_e^3 \times \mathcal{H}_o^3$;

(iii) *the external force satisfies* $f \in H_*^3(Q_{T_0})$;

(iv) (10) *and* (11) *hold;*

(v) *the function* $s \mapsto c(s)$ *is concave.*

Then, there exists $T > 0$ such that (3)–(4)–(9) admits a solution $(\rho, v) \in \mathcal{K}_T$ satisfying

$$0 < v_1 < c(\rho) \qquad \text{on } [0,T] \times \Gamma_1.$$

Moreover, if the function $s \mapsto c(s)$ satisfies $c'(s) > 0$ for all $s > 0$, then there exists $\delta > 0$ such that if $\max_{[0,T]\times\Gamma_1^i} |v_1| < \delta$ then (ρ, v) is the unique solution of (3)–(4)–(9).

Remark. If $p(\rho) = R\rho^\gamma$ with $\gamma > 1$ then $c(\rho)$ is concave for $\gamma \leq 3$; for common isentropic gases $\gamma \in (1, \frac{5}{3})$ and therefore (v) is physically meaningful. Moreover, if $\gamma > 1$ then $c'(s) > 0$ for all $s > 0$.

Remark. Uniqueness in Theorem 2 is certainly ensured in some time interval $[0, T]$ $(T > 0)$ if $\psi(0) - c(\rho_0)$ is sufficiently small on Γ_1^i (recall that by (10) we have $\psi(0) - c(\rho_0) > 0$).

3 The linearized problems in space sectors

In order to simplify notations, in the sequel we denote by $\mathbb{R}_+$ both the open interval $(0,+\infty)$ and its closure, depending on the context.

3.1 The supersonic case

In this section we first consider the case where

$$\begin{aligned} \Omega = \mathbb{R}^3_{++} &:= \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}, \\ \Gamma_1 &:= \{0\} \times \mathbb{R}_+ \times \mathbb{R}, \\ \Gamma_2 &:= \mathbb{R}_+ \times \{0\} \times \mathbb{R}, \end{aligned}$$

and we study the linearized problem

$$\begin{cases} \frac{1}{\pi}(\partial_t \rho + w \cdot \nabla \rho) + \nabla \cdot v = h & \text{in } [0,T] \times \Omega \\ \frac{\pi}{c^2(\pi)}(\partial_t v + (w \cdot \nabla) v) + \nabla \rho = k & \text{in } [0,T] \times \Omega \end{cases} \tag{12}$$

together with initial conditions (4) and boundary conditions

$$\begin{cases} (\rho, v) = (r, g) & \text{on } (0,T) \times \Gamma_1 \\ v_2 = 0 & \text{on } (0,T) \times \Gamma_2. \end{cases} \tag{13}$$

In this case $\Gamma_3 = \emptyset$ and the conditions on Γ_3 in the definitions of the spaces $\mathcal{H}^3$ disappear; in particular, we have $\mathcal{H}^3_o = \mathcal{H}^3$. Consider the set S_T of functions $(\pi, w) \in \mathcal{K}_T$ such that

$$\frac{3}{2} \sup_{\bar\Omega} \rho_0 \geq \pi \geq \frac{1}{2} \inf_{\bar\Omega} \rho_0 > 0 \qquad \text{in } \bar{Q}_T,$$

$$-w_1 + c(\pi) \leq \frac{1}{2} \sup_{\Gamma_1} (-(v_0)_1 + c(\rho_0)) < 0 \qquad \text{on } (0,T) \times \Gamma_1,$$

$$\partial_t^k \pi(0) = \partial_t^k \rho_0, \quad \partial_t^k w(0) = \partial_t^k v_0 \qquad k = 0, 1, 2,$$

where $\partial_t^k \rho_0, \partial_t^k v_0$ are the same of (8), obtained from (3), (4). From the previous assumption we infer

$$\gamma(\pi) := \min \left\{ \inf_{\bar{Q}_T} \frac{1}{\pi},\ \inf_{\bar{Q}_T} \frac{\pi}{c^2(\pi)} \right\} > 0. \tag{14}$$

Let us point out that the definition of S_T gives some restrictions on the initial data ρ_0, v_0 since some inequalities of the kind of (7) are required.

The following result holds:

Proposition 1 *Let* $\Omega = \mathbb{R}^3_{++}$, *let* $T > 0$ *and assume that:*

(i) *the boundary data* $r, g \in H^3((0,T) \times \Gamma_1)$ *satisfy*

$$(r, g) \in \mathcal{H}^3(\Gamma_1) \times \mathcal{H}^3(\Gamma_1) \times \mathcal{H}^3_e(\Gamma_1) \times \mathcal{H}^3_o(\Gamma_1) \qquad \text{for a.e. } t \in [0,T];$$

(ii) *the initial data satisfy* $(\rho_0, v_0) \in \mathcal{H}^3 \times \mathcal{H}^3 \times \mathcal{H}^3_e \times \mathcal{H}^3_o$;

(iii) *the right hand side of* (12) *satisfies* $(h, k) \in H^3(Q_T) \times H^3_*(Q_T)$, $h(t) \in \mathcal{H}^3$ *for a.e.* $t \in [0,T]$;

(iv) *the data satisfy* (7) *and the compatibility conditions* $\partial_t^k \rho(0) = \partial_t^k r(0)$, $\partial_t^k v(0) = \partial_t^k g(0)$ *on* Γ_1, $\partial_t^k v(0) \cdot \nu = 0$ *on* Γ_0, $k = 0, 1, 2$, *where* $\partial_t^k \rho(0)$, $\partial_t^k v(0)$ *are the k-th time derivatives of* ρ, v *at time* $t = 0$ *obtained from* (12), (4).

Then, for all $(\pi, w) \in S_T$ *there exists a unique* $(\rho, v) \in \mathcal{K}_T$ *solving* (12)–(4)–(13). *Moreover, there exist two constants* $C_1, C_2 > 0$ (C_1 *depends increasingly on* $|||(\pi, w)|||_{2,T}$ *while* C_2 *depends increasingly on* $|||(\pi, w)|||_{3,T}$) *such that*

$$\begin{aligned} \gamma |||(\rho, v)|||^2_{3,T} \;\le\; & \{C_1(1 + |||(\pi, w)|||^{2\varepsilon}_{3,T}) |||(\rho_0, v_0)|||^2_3 \\ & + C_1[(r, g)]^2_{3,\Sigma_T} + C_2[(h, k)]^2_{3,T}\} e^{(C_1+C_2)T}, \end{aligned} \tag{15}$$

where $\gamma = \gamma(\pi)$ *is defined in* (14) *and* $\varepsilon \in (\frac{1}{2}, 1)$.

Proof. Let $\Omega' = \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}$ and $\Gamma' = \partial\Omega'$. For all $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ we denote $\bar{x} = (x_1, -x_2, x_3)$ and for all continuous function ϕ defined on $\overline{\Omega}$ we define the functions $\tilde{\phi}$ and $\hat{\phi}$ on Ω' by

$$\tilde{\phi}(x) = \begin{cases} \phi(x) & \text{if } x_2 \ge 0 \\ \phi(\bar{x}) & \text{if } x_2 < 0 \end{cases} \qquad \hat{\phi}(x) = \begin{cases} \phi(x) & \text{if } x_2 \ge 0 \\ -\phi(\bar{x}) & \text{if } x_2 < 0; \end{cases}$$

this definition implies that the functions $\tilde{\phi}$ and $\hat{\phi}$ are, respectively, even and odd with respect to x_2: to be precise, $\hat{\phi}$ is odd only if $\phi(x_1, 0, x_3) = 0$ for all x_1, x_3. Consider now the auxiliary problem

$$\begin{cases} \frac{1}{\bar{\pi}}(\partial_t \rho + \bar{w} \cdot \nabla \rho) + \nabla \cdot v = \bar{h} & \text{in } [0,T] \times \Omega' \\ \frac{\bar{\pi}}{c^2(\bar{\pi})}(\partial_t v + (\bar{w} \cdot \nabla) v) + \nabla \rho = \bar{k} & \text{in } [0,T] \times \Omega' \\ \rho = \bar{r} \quad v = \bar{g} & \text{on } (0,T) \times \Gamma' \\ \rho(0, x) = \bar{\rho}_0(x) & \text{in } \Omega' \\ v(0, x) = \bar{v}_0(x) & \text{in } \Omega', \end{cases} \tag{16}$$

where we have set

$$\bar{\pi} = \tilde{\pi} \qquad \bar{g} = (\bar{g}_1, \bar{g}_2, \bar{g}_3) = (\tilde{g}_1, \hat{g}_2, \tilde{g}_3) \qquad \bar{r} = \tilde{r} \qquad \bar{h} = \tilde{h}$$

$$\bar{k} = (\bar{k}_1, \bar{k}_2, \bar{k}_3) = (\tilde{k}_1, \hat{k}_2, \tilde{k}_3) \qquad \bar{w} = (\bar{w}_1, \bar{w}_2, \bar{w}_3) = (\tilde{w}_1, \hat{w}_2, \tilde{w}_3)$$

$$\bar{\rho}_0 = \tilde{\rho}_0 \qquad \bar{v}_0 = ((\bar{v}_0)_1, (\bar{v}_0)_2, (\bar{v}_0)_3) = ((\tilde{v}_0)_1, (\hat{v}_0)_2, (\tilde{v}_0)_3):$$

for simplicity, we have omitted to underline the dependence on t.

Since $(\pi, w) \in \mathcal{K}_T$, it is not difficult to verify that the "reflected" functions $\bar{\pi}, \bar{w}_i$ $(i = 1, 2, 3)$ belong to $\mathcal{C}_T(H^3(\Omega'))$; moreover, (14) holds for $\bar{\pi}$ as $\gamma(\bar{\pi}) = \gamma(\pi)$. Note also that since $(\pi, w) \in S_T$ we have

$$\inf_{(0,T)\times\Gamma'} (w_1 - c(\pi)) > 0,$$

so that Γ' is noncharacteristic. Moreover we observe that $(\bar{\rho}_0, \bar{v}_0) \in H^3(\Omega')$, $(\bar{h}, \bar{k}) \in H^3((0,T) \times \Omega')$ and that $(\bar{r}, \bar{g}) \in H^3((0,T) \times \Gamma')$. Finally we observe that the compatibility conditions of order 2 hold on Γ'. We apply Theorem A.1 in [12] (also valid for halfspaces) in the case $m = s = 3$ and find a unique solution $(\rho, v) \in [\mathcal{C}_T(H^3(\Omega'))]^4$ of (16); furthermore, from the estimates (A.5) and (A.7) in [12] and by reasoning as in the proof of Lemma 3.4 in [12] we arrive at

$$\begin{aligned}\gamma |||(\rho, v)(t)|||_3^2 \ \le\ & C_1(1 + |||(\pi, w)|||_{3,T}^{2\varepsilon}) |||(\rho_0, v_0)|||_3^2 \\ & + C_1[(r, g)]_{3,\Sigma_T}^2 + C_2[(h, k)]_{3,T}^2 + (C_1 + C_2)[(\rho, v)]_{3,t}^2 :\end{aligned}$$

then, (15) follows by the Gronwall Lemma.

We claim that ρ, v_1 and v_3 are even with respect to x_2 while v_2 is odd with respect to x_2. To this end, define the functions

$$\bar{\rho}(t, x) = \rho(t, \bar{x}) \qquad \bar{v}_{1,3}(t, x) = v_{1,3}(t, \bar{x}) \qquad \bar{v}_2(t, x) = -v_2(t, \bar{x})$$

for $x \in \Omega'$; it is not difficult to verify that the couple $(\bar{\rho}, \bar{v})$ satisfies (16) as well: then, by uniqueness of the solution of (16), we infer that $\bar{\rho} \equiv \rho$ and $\bar{v} \equiv v$ in Ω', which proves the claim. Therefore, for all $t \in [0, T]$ the functions ρ, v_1, v_3 belong to the space $\mathcal{H}_o^3$ while $v_2 \in \mathcal{H}_e^3$; in particular, we have $v_2 = 0$ on $\Gamma_2 \times (0, T)$: this proves that $(\rho, v) \in \mathcal{K}_T$ solves (12)–(4)–(13). $\square$

Next we consider the case where

$$\Omega = \mathbb{R}^3_{+++} := [\mathbb{R}_+]^3, \quad \Gamma_1 := \{0\} \times \mathbb{R}_+ \times \mathbb{R}_+,$$

$$\Gamma_2 := \mathbb{R}_+ \times \{0\} \times \mathbb{R}_+, \quad \Gamma_3 := \mathbb{R}_+ \times \mathbb{R}_+ \times \{0\},$$

and we study (12)–(4) together with the boundary conditions

$$\begin{cases} (\rho, v) = (r, g) & \text{on } (0, T) \times \Gamma_1 \\ v_2 = 0 & \text{on} (0, T) \times \Gamma_2 \\ v_3 = 0 & \text{on } (0, T) \times \Gamma_3 : \end{cases} \tag{17}$$

by using the same "reflection technique" as in the proof of Proposition 1 we prove existence and uniqueness of a solution:

Proposition 2 *Let* $\Omega = \mathbb{R}^3_{+++}$*, let* $T > 0$ *and assume that* (i)–(iv) *of Proposition* 1 *hold. Then, for all* $(\pi, w) \in S_T$ *there exists a unique* $(\rho, v) \in \mathcal{K}_T$ *solving* (12)–(4)–(17)*. Moreover, there exist two constants* $C_1, C_2 > 0$ (*as in Proposition* 1) *such that* (15) *holds.*

Proof. Let $\Omega' = \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}$, $\Gamma'_1 = \{0\} \times \mathbb{R}_+ \times \mathbb{R}$ and $\Gamma'_2 = \mathbb{R}_+ \times \{0\} \times \mathbb{R}$ so that $\Gamma'_1 \bigcup \Gamma'_2 = \partial\Omega'$. For all $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ we define $\bar{x} = (x_1, x_2, -x_3)$ and for all function ϕ defined on $\bar{\Omega}$ we define the functions $\tilde{\phi}$ and $\hat{\phi}$ on Ω' by

$$\tilde{\phi}(x) = \begin{cases} \phi(x) & \text{if } x_3 \geq 0 \\ \phi(\bar{x}) & \text{if } x_3 < 0 \end{cases} \qquad \hat{\phi}(x) = \begin{cases} \phi(x) & \text{if } x_3 \geq 0 \\ -\phi(\bar{x}) & \text{if } x_3 < 0; \end{cases}$$

then, the functions $\tilde{\phi}$ and $\hat{\phi}$ are, respectively, even and odd with respect to x_3.

Consider now the auxiliary problem

$$\begin{cases} \frac{1}{\bar{\pi}}(\partial_t \rho + \bar{w} \cdot \nabla \rho) + \nabla \cdot v = \bar{h} & \text{in } [0,T] \times \Omega' \\ \frac{\bar{\pi}}{c^2(\bar{\pi})}(\partial_t v + (\bar{w} \cdot \nabla) v) + \nabla \rho = \bar{k} & \text{in } [0,T] \times \Omega' \\ \rho = \bar{r} \quad v = \bar{g} & \text{on } (0,T) \times \Gamma'_1 \\ v_2 = 0 & \text{on } (0,T) \times \Gamma'_2 \\ \rho(0,x) = \bar{\rho}_0(x) & \text{in } \Omega' \\ v(0,x) = \bar{v}_0(x) & \text{in } \Omega', \end{cases} \tag{18}$$

where we have set

$$\bar{\pi} = \tilde{\pi} \qquad \bar{g} = (\bar{g}_1, \bar{g}_2, \bar{g}_3) = (\tilde{g}_1, \tilde{g}_2, \hat{g}_3) \qquad \bar{r} = \tilde{r} \qquad \bar{h} = \tilde{h}$$
$$\bar{k} = (\bar{k}_1, \bar{k}_2, \bar{k}_3) = (\tilde{k}_1, \tilde{k}_2, \hat{k}_3) \qquad \bar{w} = (\bar{w}_1, \bar{w}_2, \bar{w}_3) = (\tilde{w}_1, \tilde{w}_2, \hat{w}_3)$$
$$\bar{\rho}_0 = \tilde{\rho}_0 \qquad \bar{v}_0 = ((\bar{v}_0)_1, (\bar{v}_0)_2, (\bar{v}_0)_3) = ((\tilde{v}_0)_1, (\tilde{v}_0)_2, (\hat{v}_0)_3).$$

Again, it is not difficult to verify that the reflected functions $(\bar{\pi}, \bar{w})$ belong to $\mathcal{C}_T(\mathcal{H}^3_*(\Omega'))$ and satisfy the properties (i)–(iv) of Proposition 1 for the reflected data; by Proposition 1, problem (18) admits a unique solution $(\rho, v) \in [\mathcal{C}_T(H^3(\Omega'))]^4$ which satisfies the estimate (15). By proceeding as in the proof of Proposition 1 we obtain that ρ, v_1 and v_2 are even with respect to x_3 while v_3 is odd with respect to x_3. Therefore, $(\rho, v) \in \mathcal{K}_T$: in particular, we have $v_3 = 0$ on $(0,T) \times \Gamma_3$ and (ρ, v) solves (12)–(4)–(17). □

3.2 The subsonic case

We repeat the same arguments of the previous section and we maintain the same notations. Again, we first consider the case where $\Omega = \mathbb{R}^3_{++}$ and we study the linearized problem (12) together with the initial conditions (4) (ρ_0 and v_0 satisfy some of the inequalities in (10) due to the definition of S_T below) and the (linear) boundary conditions

$$\begin{cases} v_2 = g_2, \quad v_3 = g_3, \quad v_1 + \rho\frac{c(\pi)}{\pi} = \psi & \text{on } (0,T)\times\Gamma_1 \\ v_2 = 0 & \text{on } (0,T)\times\Gamma_2, \end{cases} \tag{19}$$

where π is a given function; for these linearized boundary conditions the compatibility conditions read

$$\begin{aligned} &\partial_t^k v_2(0) = \partial_t^k g_2(0),\ \partial_t^k v_3(0) = \partial_t^k g_3(0), \\ &\partial_t^k v_1(0) + \partial_t^k\left(\rho\tfrac{c(\pi)}{\pi}\right)(0) = \partial_t^k \psi(0) \text{ on } \Gamma_1 \\ &\partial_t^k v(0)\cdot\nu = 0 \qquad \text{on } \Gamma_0 \qquad k=0,1,2, \end{aligned} \tag{20}$$

where $\partial_t^k\rho(0), \partial_t^k v(0)$ are the k-th time derivatives of ρ, v at time $t=0$, obtained from (12), (4). Consider now the set S_T of functions $(\pi, w)\in\mathcal{K}_T$ which satisfy

$$\begin{gathered} \frac{3}{2}\sup_{\bar\Omega}\rho_0 \geq \pi \geq \frac{1}{2}\inf_{\bar\Omega}\rho_0 > 0 \qquad \text{in } \bar{Q}_T \\ -w_1 + c(\pi) \geq \frac{1}{2}\inf_{\Gamma_1}(-(v_0)_1 + c(\rho_0)) > 0 \qquad \text{on } (0,T)\times\Gamma_1 \\ w_1 \geq \frac{1}{2}\inf_{\Gamma_1}(v_0)_1 > 0 \qquad \text{on } (0,T)\times\Gamma_1 \\ \partial_t^k\pi(0) = \partial_t^k\rho_0, \quad \partial_t^k w(0) = \partial_t^k v_0 \qquad k=0,1,2. \end{gathered}$$

The following result holds:

Proposition 3 *Let $\Omega = \mathbb{R}^3_{++}$, let $T>0$ and assume that:*

(i) $\psi, g_2, g_3 \in H^3((0,T)\times\Gamma_1)$ *satisfy*

$$g_2 \in \mathcal{H}^3_e(\Gamma_1), \quad g_3 \in \mathcal{H}^3_o(\Gamma_1), \quad \psi\in\mathcal{H}^3(\Gamma_1) \qquad \textit{for a.e. } t\in[0,T];$$

(ii) *the initial data satisfy* $(\rho_0, v_0)\in\mathcal{H}^3\times\mathcal{H}^3\times\mathcal{H}^3_e\times\mathcal{H}^3_o$;

(iii) *the right hand side of* (12) *satisfies* $(h,k)\in H^3(Q_T)\times H^3_*(Q_T)$, $h(t)\in\mathcal{H}^3$ *for a.e.* $t\in[0,T]$;

(iv) *the data satisfy* $(10)_1, (10)_2$ *and the compatibility conditions* (20) *of order* 2.

Then, for all $(\pi,w)\in S_T$ there exists a unique $(\rho,v)\in\mathcal{K}_T$ solving (12)–(4)–(19). Moreover, there exist three constants $C_1, C_2, C_M > 0$ (C_1 depends increasingly *on* $|||(\pi, w)|||_{2,T}$*,* C_2 *depends increasingly on* $|||(\pi, w)|||_{3,T}$*,* C_M *depends increasingly on* $[(\psi, g_2, g_3)]_{3,\Sigma_T}$*) such that*

$$|||(\rho, v)(t)|||_3^2 + [(\rho, v)]^2_{3,\Sigma_t} \leq C_1(1 + |||(\pi, w)|||^{2\varepsilon}_{3,T})|||(\rho_0, v_0)|||_3^2$$
$$+C_1[(\psi, g_2, g_3)]^2_{3,\Sigma_t} + C_2[(h, k)]^2_{3,t} + (C_1 C_M + C_2)[(\rho, v)]^2_{3,t} \tag{21}$$

for all $t \in [0, T]$*; here,* $\varepsilon \in (\frac{1}{2}, 1)$*.*

Proof. Let $\Omega' = \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}$, $\Gamma' = \partial\Omega'$ and consider the auxiliary problem

$$\begin{cases} \frac{1}{\bar\pi}(\partial_t \rho + \bar w \cdot \nabla\rho) + \nabla \cdot v = \bar h & \text{in } [0,T] \times \Omega' \\ \frac{\bar\pi}{c^2(\bar\pi)}(\partial_t v + (\bar w \cdot \nabla)v) + \nabla\rho = \bar k & \text{in } [0,T] \times \Omega' \\ v_2 = \bar g_2, \quad v_3 = \bar g_3, \quad v_1 + \rho\frac{c(\bar\pi)}{\bar\pi} = \bar\psi & \text{on } (0,T) \times \Gamma' \\ \rho(0,x) = \bar\rho_0(x) & \text{in } \Omega' \\ v(0,x) = \bar v_0(x) & \text{in } \Omega', \end{cases} \tag{22}$$

where we have set

$$\bar\pi = \tilde\pi \qquad (\bar g_2, \bar g_3) = (\hat g_2, \tilde g_3) \qquad \bar\psi = \tilde\psi \qquad \bar h = \tilde h$$
$$\bar k = (\bar k_1, \bar k_2, \bar k_3) = (\tilde k_1, \hat k_2, \tilde k_3) \qquad \bar w = (\bar w_1, \bar w_2, \bar w_3) = (\tilde w_1, \hat w_2, \tilde w_3)$$
$$\bar\rho_0 = \tilde\rho_0 \qquad \bar v_0 = ((\bar v_0)_1, (\bar v_0)_2, (\bar v_0)_3) = ((\tilde v_0)_1, (\hat v_0)_2, (\tilde v_0)_3).$$

Note that Γ' is noncharacteristic and by Theorem A.1 in [12] we find a unique solution $(\rho, v) \in [\mathcal{C}_T(H^3(\Omega'))]^4$ of (22); moreover, we get (21) by (A.5) and (A.7) in [12] and by arguing as in the proof of Lemma 3.4 in the same paper.

Since also $\rho(t, \bar x), v_{1,3}(t, \bar x), -v_2(t, \bar x)$ solves (22), by uniqueness of the solution of (22) we infer that ρ, v_1 and v_3 are even with respect to x_2 while v_2 is odd with respect to x_2: then $(\rho, v) \in \mathcal{K}_T$ solves (12)–(4)–(19). □

Remark. In (21) we do not highlight the dependence of the constants C_1, C_2, C_M on γ (defined in (14)) because γ is uniformly bounded from below and above when (π, w) vary in S_T.

Next, consider the case where $\Omega = \mathbb{R}^3_{+++}$ and the problem (12)–(4) together with the boundary conditions

$$\begin{cases} v_2 = g_2, \quad v_3 = g_3, \quad v_1 + \rho\frac{c(\pi)}{\pi} = \psi & \text{on } (0,T) \times \Gamma_1 \\ v_2 = 0 & \text{on } (0,T) \times \Gamma_2 \\ v_3 = 0 & \text{on } (0,T) \times \Gamma_3; \end{cases} \tag{23}$$

then, we obtain the following:

Proposition 4 *Let* $\Omega = \mathbb{R}^3_{+++}$*, let* $T > 0$ *and assume that* (i)–(iv) *of Proposition* 3 *hold.*

Then, for all $(\pi, w) \in S_T$ *there exists a unique* $(\rho, v) \in \mathcal{K}_T$ *solving* (12)–(4)–(23)*. Moreover, there exist three constants* $C_1, C_2, C_M > 0$ (*as in Proposition* 3) *such that* (21) *holds for all* $t \in [0, T]$.

Proof. It follows by the same arguments used in the proof of Proposition 2. □

Finally, we also need a similar result for the subsonic outflow problem: here we take $\Omega = (-\infty, 2) \times \mathbb{R}_+ \times \mathbb{R}_+$ instead of $\mathbb{R}^3_{+++}$ so that we do not have to change sign to the functions defined on $\partial\Omega$. Let $\Gamma_1 = \{2\} \times \mathbb{R}_+ \times \mathbb{R}_+$, $\Gamma_2 = (-\infty, 2] \times \{0\} \times \mathbb{R}_+$, $\Gamma_3 = (-\infty, 2] \times \mathbb{R}_+ \times \{0\}$ and consider the boundary conditions

$$\begin{cases} v_1 = g_1 & \text{on } (0,T) \times \Gamma_1 \\ v_2 = 0 & \text{on } (0,T) \times \Gamma_2 \\ v_3 = 0 & \text{on } (0,T) \times \Gamma_3. \end{cases} \tag{24}$$

Consider the same set S_T as for Proposition 3; then, we obtain the following:

Proposition 5 *Let* $\Omega = (-\infty, 2) \times \mathbb{R}_+ \times \mathbb{R}_+$*, let* $T > 0$ *and assume that:*

(i) $g_1 \in H^3((0,T) \times \Gamma_1)$ *satisfies* $g_1 \in \mathcal{H}^3(\Gamma_1)$ *for a.e.* $t \in [0, T]$*;*

(ii) *the initial data satisfy* $(\rho_0, v_0) \in \mathcal{H}^3 \times \mathcal{H}^3 \times \mathcal{H}^3_e \times \mathcal{H}^3_o$*;*

(iii) *the right hand side of* (12) *satisfies* $(h, k) \in H^3(Q_T) \times H^3_*(Q_T)$, $h(t) \in \mathcal{H}^3$ *for a.e.* $t \in [0, T]$*;*

(iv) *the data satisfy* $(10)_1, (10)_3, (10)_4$ *and the compatibility conditions of order* 2.

Then, for all $(\pi, w) \in S_T$ *there exists a unique* $(\rho, v) \in \mathcal{K}_T$ *solving* (12)–(4)–(24)*. Moreover, there exist three constants* $C_1, C_2 > 0$ (*as in Proposition* 3) *and* $C_M > 0$ (*depending increasingly on* $[g_1]_{3,\Sigma_T}$) *such that* (21) *holds* (*with* $[g_1]_{3,\Sigma_t}$ *instead of* $[(\psi, g_2, g_3)]_{3,\Sigma_t}$) *for all* $t \in [0, T]$.

Proof. It can be obtained by "double reflection" as for Propositions 1 and 2. □

4 Proof of theorem 1

Let Ω be the bounded cylinder defined in (2), let S_T be the subset of $\mathcal{K}_T$ introduced in Section 3.1; we prove existence and uniqueness for the linearized problem in $(0, T) \times \Omega$:

$$\begin{cases} \frac{1}{\pi}(\partial_t \rho + w \cdot \nabla \rho) + \nabla \cdot v = 0 & \text{in } [0,T] \times \Omega \\ \frac{\pi}{c^2(\pi)}(\partial_t v + (w \cdot \nabla) v - f) + \nabla \rho = 0 & \text{in } [0,T] \times \Omega \end{cases} \tag{25}$$

together with initial conditions (4) and boundary conditions (6).

Proposition 6 *Let $\Omega \subset \mathbb{R}^3$ be as in* (2)*, let $T > 0$ and assume that* (i)–(iv) *of Theorem* 1 *hold* (*with T_0 replaced by T*)*. Then, for all $(\pi, w) \in S_T$ there exists a unique $(\rho, v) \in \mathcal{K}_T$ solving* (25)–(4)–(6)*. Moreover, there exist two constants $C_1, C_2 > 0$* (*C_1 depends increasingly on $|||(\pi, w)|||_{2,T}$ while C_2 depends increasingly on $|||(\pi, w)|||_{3,T}$*) *such that*

$$\begin{aligned}\gamma |||(\rho, v)|||^2_{3,T} \;\leq\; & \{C_1(1 + |||(\pi, w)|||^{2\varepsilon}_{3,T})|||(\rho_0, v_0)|||^2_3 \\ & + C_1[(r, g)]^2_{3,\Sigma_T} + C_2[f]^2_{3,T}\} e^{(C_1+C_2)T},\end{aligned} \tag{26}$$

where $\gamma = \gamma(\pi)$ is defined in (14) *and $\varepsilon \in (\frac{1}{2}, 1)$.*

Proof. We first prove the result in the unbounded cylinder

$$\Omega' = (0, +\infty) \times (0, 1) \times (0, 1)$$

and in the time interval $[0, T]$. We localize (12) in Ω' by using a partition of unity for $\bar{\Omega}'$ so that the problem reduces to four problems considered in the previous section: cover $\bar{\Omega}'$ by a family of 4 open sets $\{\mathcal{U}_i\}$ $(i = 1, \ldots, 4)$ so that each one of them contains one (and only one) of the infinite edges of Ω', namely the lines $\{x_2 = x_3 = 0\}, \{x_2 = 1, x_3 = 0\}, \{x_2 = 0, x_3 = 1\}, \{x_2 = x_3 = 1\}$; moreover, each $\mathcal{U}_i$ should not intersect the two faces of Ω' which are not adjacent to the edge contained in $\mathcal{U}_i$. Take a partition of unity $\{\chi_i\}_{i=1}^4$ subordinate to the covering $\{\mathcal{U}_i\}$ such that $\sum_{i=1}^4 \chi_i = 1, \chi_i \geq 0, \chi_i \in \mathcal{H}^3$. We multiply (25) by χ_i, $i = 1, \ldots, 4$. After a suitable change of variables, we obtain the following equations in $\mathbb{R}^3_{+++}$ for $(\rho_i, v_i) = (\chi_i \rho, \chi_i v)$:

$$\begin{cases} \frac{1}{\pi}(\partial_t \rho_i + w \cdot \nabla \rho_i) + \nabla \cdot v_i = \frac{1}{\pi} w \cdot \nabla \chi_i \rho + v \cdot \nabla \chi_i \\ \frac{\pi}{c^2(\pi)}(\partial_t v_i + (w \cdot \nabla) v_i - \chi_i f) + \nabla \rho_i = \frac{\pi}{c^2(\pi)}(w \cdot \nabla \chi_i) v + \rho \nabla \chi_i. \end{cases} \tag{27}$$

In view of a fixed point $\pi = \rho, w = v$, instead of (27) we consider the problems

$$\begin{cases} \frac{1}{\pi}(\partial_t \rho_i + w \cdot \nabla \rho_i) + \nabla \cdot v_i = 2w \cdot \nabla \chi_i & \text{in } [0, T] \times \mathbb{R}^3_{+++} \\ \frac{\pi}{c^2(\pi)}(\partial_t v_i + (w \cdot \nabla) v_i - \chi_i f) + \nabla \rho_i & \\ \quad = \frac{\pi}{c^2(\pi)}(w \cdot \nabla \chi_i) w + \pi \nabla \chi_i & \text{in } [0, T] \times \mathbb{R}^3_{+++} \\ (\rho_i, v_i) = (\chi_i r, \chi_i g) & \text{on } [0, T] \times \Gamma_1 \\ v_i \cdot \nu = 0 & \text{on } [0, T] \times \Gamma_0 \\ \rho_i(0, x) = \chi_i(x) \rho_0(x) & \text{in } \mathbb{R}^3_{+++} \\ v_i(0, x) = \chi_i(x) v_0(x) & \text{in } \mathbb{R}^3_{+++}. \end{cases} \tag{28}$$

We verify that the data of problem (28) satisfy (i)–(iv) of Proposition 2; in particular (iii) follows from $(\pi, w) \in \mathcal{C}_T(\mathcal{H}^3_*)$ and $\chi_i \in \mathcal{H}^3$, the compatibility conditions (iv) follow from $\partial_t^k \pi(0) = \partial_t^k \rho_0, \partial_t^k w(0) = \partial_t^k v_0, k = 0, 1, 2,$ and (8). By

Proposition 2, (28) admits a unique solution $(\rho_i, v_i) \in \mathcal{K}_T$. By adding together the functions (ρ_i, v_i) we obtain a solution (ρ, v) in $(0,T) \times \Omega'$ of (25), (4) and the first boundary condition on $(0,T) \times \Gamma_1^i$ of (6). As regards the boundary conditions on Γ_2 and Γ_3, if $v_i \cdot \nu \not\equiv 0$ on $\mathcal{P} = \{x_2 = 1\} \cup \{x_3 = 1\}$ for some i, after adding the solutions (ρ_i, v_i) we could obtain $v_2 \neq 0$ on Γ_2 or $v_3 \neq 0$ on Γ_3. To overcome this point, we proceed in two steps. First, since each (ρ_i, v_i) has initially a support which does not intersect $\mathcal{P}$, by using the finite speed of propagation we show that (ρ_i, v_i) vanishes on $\mathcal{P}$ for each $t \in (0, T')$, for a sufficiently small $T' > 0$. It follows that (after the inverse change of variables) $\sum_i v_i \cdot \nu = 0$ on Γ_0. Thus we have found a unique solution (ρ, v) of (25), (4), (6) defined on $[0, T'] \times \Omega'$. We verify that T' depends only on $\|w\|_{L^\infty}$ and on the extension of each support, namely on the functions χ_i. We take $t = T'$ as a new initial time, decompose the data by means of the χ_i's and by the same arguments as above find a solution defined on $[T', 2T']$. We proceed by this continuation argument up to when the solution is extended to the whole interval $[0, T]$. Moreover, since all the (ρ_i, v_i) satisfy (15), (ρ, v) satisfies (26). $\square$

For all given r, g, ρ_0, v_0, f satisfying the assumptions of Theorem 1 is defined a map Λ such that $\Lambda(\pi, w) = (\rho, v)$; we achieve the proof of Theorem 1 by showing that Λ admits a fixed point:

Lemma 1 *Assume that r, g, ρ_0, v_0, f satisfy the assumptions of Theorem 1; then, for sufficiently small T there exists a compact subset $\mathcal{S}$ of $C(0, T, L^2(\Omega))$ such that the map Λ defines a contraction in $\mathcal{S}$.*

Proof. For the moment take $T \in (0, T_0)$: if needed, later on we will take a smaller value of T. Let $u' = (\pi, w)$ and let $u = (\rho, v) = \Lambda u'$; as for (5), we may write (25) as a linear symmetric hyperbolic system, that is in the form

$$L(u')u = A_0(u')\partial_t u + \sum_{j=1}^{3} A_j(u')\partial_j u = F(u'),$$

together with the initial and boundary conditions (4) and (6) which we write as

$$\begin{aligned} &u(0,x) = u_0(x) && \text{in } \Omega \\ &Mu = G && \text{on } (0,T) \times \partial\Omega; \end{aligned}$$

in this case we have

$$\begin{array}{lll} M = Id, & G = (r, g) & \text{on } (0,T) \times \Gamma_1^i, \\ M = 0, & G = 0 & \text{on } (0,T) \times \Gamma_1^o \\ M = (0,0,1,0), & G = 0 & \text{on } (0,T) \times \Gamma_2, \\ M = (0,0,0,1), & G = 0 & \text{on } (0,T) \times \Gamma_3. \end{array}$$

Step 1 definition of the set $\mathcal{S}$.

Choose $K_1 > |||u_0|||_3$: if needed, later on we will take a larger value of K_1; choose also $K_2 > |||u_0|||_2$. Consider the set $\mathcal{S}$ of functions $u' = (\pi, w) \in \mathcal{K}_T$ such that

$$\frac{3}{2}\sup_{\bar{\Omega}} \rho_0 \geq \pi \geq \frac{1}{2}\inf_{\bar{\Omega}} \rho_0 > 0 \qquad \text{in } \bar{Q}_T$$

$$-w_1 + c(\pi) \leq \frac{1}{2}\max_{\Gamma_1}(-(v_0)_1 + c(\rho_0)) < 0 \qquad \text{on } (0,T)\times\Gamma_1$$

$$\partial_t^k u'(0) = \partial_t^k u_0 \qquad \text{in } \bar{\Omega} \qquad k = 0,1,2$$

$$|||u'|||_{3,T} \leq K_1 \qquad |||u'x|||_{2,T} \leq K_2.$$

Step 2 proof that $\mathcal{S} \neq \emptyset$.

By reasoning as in the proof of Lemma 3.2 in [12] we obtain that for all $K_1 > |||u_0|||_3$ there exists $T_{K_1} > 0$ such that for all $T \leq T_{K_1}$ we have $\mathcal{S} \neq \emptyset$.

Step 3 proof that $\mathcal{S}$ is compact in $X = C(0,T;L^2(\Omega))$.

Consider a sequence $\{u'_k\} \subset \mathcal{S}$ and note that the set $\mathcal{S}$ is bounded in $\mathcal{K}_T$: in particular, $\mathcal{S}$ is bounded in $C^1(0,T;H^2(\Omega))$. Then, by the Ascoli-Arzela Theorem and the compact imbedding $H^2(\Omega) \subset L^2(\Omega)$ we can extract a subsequence converging in X to some $u' \in X$. Furthermore, u' satisfies the inequalities which characterize $\mathcal{S}$: indeed, the pointwise inequalities in $\bar{Q}_T$ and on $(0,T)\times\Gamma_1$ are satisfied by the compact imbedding $\mathcal{K}_T \subset C(\bar{Q}_T)$ while the bounds on the norms are satisfied by the lower semicontinuity of the norms under weak* convergence. Hence, $u' \in \mathcal{S}$ and $\mathcal{S}$ is compact.

Step 4 proof that $\Lambda(\mathcal{S}) \subseteq \mathcal{S}$.

By Proposition 6 (and (26)) there exist two constants $C_1, C_2 > 0$ (C_1 depends increasingly on $|||u'|||_{2,T}$ while C_2 depends increasingly on $|||u'|||_{3,T}$) such that

$$|||u|||_{3,T}^2 \leq \{C_1(1 + |||u'|||_{3,T}^{2\varepsilon})|||u_0|||_3^2 + C_1[G]_{3,\Sigma_T}^2 + C_2[f]_{3,T}^2\}e^{(C_1+C_2)T};$$

here, the data are fixed, so that γ is given and can be included in C_1, C_2. The proof then follows by reasoning as in Lemma 3.4 in [12], the only differences being that we deal with the H^3 norm instead of the H^4 norm: in particular, by the imbedding $H^2(\Omega) \subset C(\Omega)$ we have that $\mathcal{S} \subset C^1(0,T;C(\bar{\Omega}))$ so that u satisfies the inequalities that characterize $\mathcal{S}$ on some interval of time $[0,T]$, $T > 0$.

Step 5 proof that Λ is a contraction in $\mathcal{S}$.

We argue as in [12]. Take $u_1' = (\pi_1, w_1)$, $u_2' = (\pi_2, w_2)$ in $\mathcal{S}$ and let $u_i = (\rho_i, v_i) = \Lambda u_i'$ for $i = 1, 2$: then, $u_1 - u_2$ satisfies

$$\begin{cases} L(u_1')(u_1 - u_2) = [L(u_2') - L(u_1')](u_2) + F(u_1') - F(u_2') & \text{in } [0,T] \times \Omega \\ M(u_1 - u_2) = 0 & \text{on } (0,T) \times \partial\Omega \\ (u_1 - u_2)(0, x) = 0 & \text{in } \Omega; \end{cases}$$

now, set $H = [L(u_2') - L(u_1')](u_2) + F(u_1') - F(u_2')$, multiply the first equation by $u_1 - u_2$ and integrate by parts over Ω to obtain

$$\begin{aligned} &\frac{d}{dt} \int_\Omega (A_0(u_1')(u_1 - u_2), u_1 - u_2) + \int_{\partial\Omega} (A_\nu(u_1')(u_1 - u_2), u_1 - u_2) \\ &\quad = \int_\Omega (B(u_1')(u_1 - u_2), u_1 - u_2) + 2 \int_\Omega (H, u_1 - u_2), \end{aligned}$$

where $B = \partial_t A_0 + \sum_j \partial_j A_j$. Note that $u_1 - u_2 \in \ker M$ so that, by the maximally non-negativity assumption, the boundary integral is non negative; moreover, by Hölder inequality we get the estimate

$$\int_\Omega (B(u_1')(u_1 - u_2), u_1 - u_2) \le C |||u_1 - u_2|||_{0,T}^2 :$$

hence, again Hölder inequality yields

$$\begin{aligned} &\frac{d}{dt} \int_\Omega (A_0(u_1')(u_1 - u_2), u_1 - u_2) \\ &\quad \le C |||u_1 - u_2|||_{0,T}^2 + 2\|H(t)\| \, \|u_1'(t) - u_2'(t)\|. \end{aligned}$$

Finally, integrate over $[0, t]$, note that $[H]_{0,t} \le C[u_1' - u_2']_{0,t}$, take into account the positive definiteness of A_0 (say $A_0 \ge \gamma$) and use Young inequality to obtain

$$\gamma |||u_1 - u_2|||_{0,T}^2 \le C_1 T |||u_1 - u_2|||_{0,T}^2 + C_2 T |||u_1' - u_2'|||_{0,T}^2 ;$$

now take $T \le \frac{\gamma}{2C_1}$ so that the previous inequality becomes

$$|||u_1 - u_2|||_{0,T}^2 \le CT |||u_1' - u_2'|||_{0,T}^2 :$$

if we take $CT < 1$, then Λ is a contraction in $\mathcal{S}$ with respect to the $C(0, T; L^2(\Omega))$ norm. $\square$

We are now ready to give the

Proof of Theorem 1. If T is small enough, Lemma 1 states that Λ has a unique fixed point $(\rho, v) \in \mathcal{S}$: by definition of $\mathcal{S}$ and by Proposition 6, $(\rho, v) \in \mathcal{K}_T$ solves (3)–(4)–(6) and satisfies $v_1 > c(\rho)$ on $(0, T) \times \Gamma_1^o$.

5 Proof of theorem 2

The proof follows the same lines as that of Theorem 1; nevertheless, it is slightly more delicate since nonlinear boundary conditions are involved. Let Ω be as in (2) and consider the boundary conditions

$$\begin{cases} v_2 = g_2, \quad v_3 = g_3, \quad v_1 + \rho\frac{c(\pi)}{\pi} = \psi & \text{on } (0,T)\times\Gamma_1^i \\ v_1 = g_1 & \text{on } (0,T)\times\Gamma_1^o \\ v_2 = 0 & \text{on } (0,T)\times\Gamma_2 \\ v_3 = 0 & \text{on } (0,T)\times\Gamma_3. \end{cases} \tag{29}$$

Consider the same set S_T as for Proposition 3; by reasoning as for Proposition 6, we obtain

Proposition 7 *Let $\Omega \subset \mathbb{R}^3$ be as in* (2)*, let $T > 0$ and assume that:*

(i) $\psi, g_2, g_3 \in H^3((0,T)\times\Gamma_1^i)$, $g_1 \in H^3((0,T)\times\Gamma_1^o)$ *satisfy*

$$g_2 \in \mathcal{H}_e^3(\Gamma_1^i),\ \ g_3 \in \mathcal{H}_o^3(\Gamma_1^i),\ \ \psi \in \mathcal{H}^3(\Gamma_1^i),\ \ g_1 \in \mathcal{H}^3(\Gamma_1^o)\ \ \textit{for a.e. } t\in[0,T];$$

(ii) *the initial data satisfy* $(\rho_0, v_0) \in \mathcal{H}^3 \times \mathcal{H}^3 \times \mathcal{H}_e^3 \times \mathcal{H}_o^3$*;*

(iii) *the external force satisfies* $f \in H_*^3(Q_T)$*;*

(iv) *the data satisfy* (10) *and the compatibility conditions*

$$\begin{gathered} (\partial_t^k v_0)_2 = \partial_t^k g_2(0),\ (\partial_t^k v_0)_3 = \partial_t^k g_3(0), \\ (\partial_t^k v_0)_1 + \partial_t^k\left(\rho_0\frac{c(\pi(0))}{\pi(0)}\right) = \partial_t^k\psi(0)\ \textit{ on } \Gamma_1^i \\ (\partial_t^k v_0)_1 = \partial_t^k g_1(0) \qquad \textit{on } \Gamma_1^o \\ \partial_t^k v_0\cdot\nu = 0 \qquad \textit{on } \Gamma_0 \qquad k=0,1,2. \end{gathered}$$

Then, for all $(\pi, w) \in S_T$ there exists a unique $(\rho, v) \in \mathcal{K}_T$ solving (25)–(4)–(29)*. Moreover, there exist three constants $C_1, C_2, C_M > 0$ (C_1 depends increasingly on $|||(\pi,w)|||_{2,T}$, C_2 depends increasingly on $|||(\pi,w)|||_{3,T}$, C_M depends increasingly on $[(\psi, g_1, g_2, g_3)]_{3,\Sigma_T}$) such that*

$$\begin{aligned} |||(\rho,v)(t)|||_3^2 + [(\rho,v)]_{3,\Sigma_t}^2 &\le C_1(1+|||(\pi,w)|||_{3,T}^{2\varepsilon})|||(\rho_0,v_0)|||_3^2 \\ &+ C_1[(\psi,g_1,g_2,g_3)]_{3,\Sigma_t}^2 + C_2[f]_{3,t}^2 + (C_1C_M + C_2)[(\rho,v)]_{3,t}^2 \end{aligned} \tag{30}$$

for all $t \in [0,T]$; here, $\varepsilon \in (\frac{1}{2}, 1)$.

Proof. The partition of unity is slightly different from that in Proposition 6: cover $\bar{\Omega}$ by a family of 8 open sets $\{\mathcal{U}_i\}$ $(i = 1,\dots,8)$ so that each one of them contains one (and only one) of the vertices of Ω, namely the points $V_1(0,0,0)$, $V_2(0,1,0)$, $V_3(0,0,1)$, $V_4(0,1,1)$, $V_5(2,0,0)$, $V_6(2,1,0)$, $V_7(2,0,1)$, $V_8(2,1,1)$; moreover, each $\mathcal{U}_i$ should not intersect the three faces of $\bar{\Omega}$ which are not adjacent to the vertex contained in $\mathcal{U}_i$. Then, for $i = 1, \dots, 4$, (25)–(4)–(29) reduces to the problem (28) with subsonic linearized boundary conditions, solved by Proposition 4, while for $i = 5, \dots, 8$ it reduces to a subsonic outflow problem that we solve by Proposition 5. Then, the proof follows. $\square$

For all given ψ, g, ρ_0, v_0, f satisfying the assumptions of Proposition 7 is defined a map Λ such that $\Lambda(\pi, w) = (\rho, v)$; in order to prove a result similar to Lemma 1 in the subsonic case we need to take into account the boundary conditions:

Lemma 2 *Assume that ψ, g, ρ_0, v_0, f satisfy the assumptions of Proposition 7; then, for sufficiently small T there exists a compact subset $\mathcal{S}$ of $C(0,T,L^2(\Omega)) \cap L^2(\Sigma_T)$ such that Λ maps continuously $\mathcal{S}$ into itself.*

Proof. To start, just take $T \in (0, T_0]$: if necessary, later on we will take a smaller value of T.

Let $u' = (\pi, w)$ and let $u = (\rho, v) = \Lambda u'$; we write (25) in the form

$$L(u')u = A_0(u')\partial_t u + \sum_{j=1}^{3} A_j(u')\partial_j u = F(u'),$$

together with the initial and boundary conditions (4) and (29) which we write as

$$\begin{aligned} &u(0,x) = u_0(x) && \text{in } \Omega \\ &M(u')u = G && \text{on } (0,T) \times \partial\Omega; \end{aligned}$$

in this case we have

$$M(u') = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{c(\pi)}{\pi} & 1 & 0 & 0 \end{pmatrix}, \quad G = (g_2, g_3, \psi) \quad \text{on} (0,T) \times \Gamma_1^i,$$

$$\begin{aligned} &M = (0,1,0,0), && G = g_1 \ \text{ on } (0,T) \times \Gamma_1^o, \\ &M = (0,0,1,0), && G = 0 \ \text{ on } (0,T) \times \Gamma_2, \\ &M = (0,0,0,1), && G = 0 \ \text{ on } (0,T) \times \Gamma_3. \end{aligned}$$

Step 1 definition of the set $\mathcal{S}$.

Choose $K_1 > 0$ so that $K_1 > |||u_0|||_3$: if needed, later on we will take a larger value of K_1; choose also $K_2 > |||u_0|||_2$. Consider the set $\mathcal{S}$ of functions $u' = (\pi, w) \in \mathcal{K}_T$ such that

$$\frac{3}{2}\sup_{\bar\Omega}\rho_0 \geq \pi \geq \frac{1}{2}\inf_{\bar\Omega}\rho_0 > 0 \qquad \text{in } \bar Q_T$$

$$-w_1 + c(\pi) \geq \frac{1}{2}\min_{\Gamma_1}(-(v_0)_1 + c(\rho_0)) > 0 \qquad \text{on } (0,T)\times\Gamma_1$$

$$w_1 \geq \frac{1}{2}\min_{\Gamma_1}(v_0)_1 > 0 \qquad \text{on } (0,T)\times\Gamma_1$$

$$\partial_t^k u'(0) = \partial_t^k u_0 \qquad \text{in } \bar\Omega \qquad k = 0,1,2$$

$$[u']_{3,\Sigma_T} \leq K_1 \qquad |||u'|||_{3,T} \leq K_1 \qquad |||u'|||_{2,T} \leq K_2.$$

Step 2 proof that $\mathcal{S} \neq \emptyset$.

This can be obtained as in Step 2 in the proof of Lemma 1.

Step 3 proof that $\mathcal{S}$ is compact in $X = C(0,T;L^2(\Omega)) \cap L^2(\Sigma_T)$.

Consider a sequence $\{u'_k\} \subset \mathcal{S}$: by Step 3 in the proof of Lemma 1 we know that it converges in $C(0,T;L^2(\Omega))$ to some $u \in \mathcal{S}$, up to a subsequence. Moreover, $u_k \to u'$ in $L^2(\Sigma_T)$ and hence $\mathcal{S}$ is compact.

Step 4 proof that $\Lambda(\mathcal{S}) \subseteq \mathcal{S}$.

By Proposition 7 (and (30)) there exist three constants $C_1, C_2, C_M > 0$ (C_1 depends increasingly on $|||u'|||_{2,T}$, C_2 depends increasingly on $|||u'|||_{3,T}$, C_M depends increasingly on $[u']_{3,\Sigma_T}$) such that

$$\begin{aligned} |||u(t)|||_3^2 + [u]_{3,\Sigma_t}^2 \;\leq\;& C_1(1 + |||u'|||_{3,T}^{2\varepsilon})|||u_0|||_3^2 \\ & + C_1[G]_{3,\Sigma_t}^2 + C_2[f]_{3,t}^2 + (C_1C_M + C_2)[u]_{3,t}^2 \end{aligned}$$

for all $t \in [0,T]$. The proof then follows by reasoning as in Lemma 3.4 in [12] with the H^4 norms replaced by the H^3 norms.

Step 5 proof that Λ is continuous in $\mathcal{S}$.

We argue as in the proof of Lemma 3.5 in [12]. Let $u'_1, u'_2 \in \mathcal{S}$ and let $u_i = \Lambda u'_i$ ($i = 1,2$); consider the two problems corresponding to u_i and u'_i ($i = 1,2$), subtract them and multiply by $u_1 - u_2$: then, with an integration by parts on Ω we get

$$\begin{aligned} &\frac{d}{dt}\int_\Omega (A_0(u'_1)(u_1-u_2), u_1-u_2) + \int_{\partial\Omega}(A_\nu(u'_1)(u_1-u_2), u_1-u_2) \\ &= \int_\Omega (B(u'_1)(u_1-u_2), u_1-u_2) + 2\int_\Omega (H, u_1-u_2), \end{aligned}$$

where $H = [L(u'_2) - L(u'_1)](u_2) + F(u'_1) - F(u'_2)$ and $B = \partial_t A_0 + \sum_j \partial_j A_j$. Integrate the previous equality on $[0,t]$ and take into account the estimates

$$[M(u'_2)u_2 - M(u'_1)u_2]_{0,\Sigma_t} \leq c[u'_1 - u'_2]_{0,\Sigma_t} \qquad [H]_{0,t} \leq c[u'_1 - u'_2]_{0,t}$$

to obtain

$$\begin{aligned} &\|(u_1 - u_2)(t)\|^2 + [u_1 - u_2]^2_{0,\Sigma_t} \\ &\quad \leq c([u'_1 - u'_2]^2_{0,\Sigma_t} + [u'_1 - u'_2]^2_{0,t} + [u_1 - u_2]^2_{0,t}); \end{aligned}$$

the continuity of Λ follows by applying the Gronwall Lemma and from the estimate $[u'_1 - u'_2]^2_{0,T} \leq T|||u'_1 - u'_2|||^2_{0,T}$. □

We are now ready to give the

Proof of Theorem 2. As $\rho \mapsto c(\rho)$ is concave, the set $\mathcal{S}$ found in Lemma 2 is convex; therefore, by the Schauder fixed point theorem, the map Λ admits a fixed point $(\rho, v) \in \mathcal{S}$; by definition of $\mathcal{S}$ and by Proposition 7, $(\rho, v) \in \mathcal{K}_T$ solves (3)–(4)–(9) and satisfies $0 < v_1 < c(\rho)$ on $[0,T] \times \Gamma_1$.

Assume now that there exists $\delta > 0$ such that $\max_{[0,T]\times\Gamma_1^i} |v_1| < \delta$; then uniqueness follows by arguing as in the proof of Theorem 2 in [12].

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References

[1] R. AGEMI, The initial boundary value problem for *inviscid* barotropic fluid motion, *Hokkaido Math. J.* **10** (1981), 156–182.

[2] H. BEIRÃO DA VEIGA, On the barotropic motion of compressible perfect fluids, *Ann. Sc. Norm. Sup. Pisa*, **8** (1981), 317–351.

[3] H. BEIRÃO DA VEIGA, Perturbation theorems for linear hyperbolic mixed problems and applications to the compressible Euler equations, *Comm. Pure Appl. Math.* **46** (1993), 221–259.

[4] K.O. FRIEDRICHS, Symmetric positive linear differential equations, *Comm. Pure Appl. Math.* **11** (1958), 333–418.

[5] P. GRISVARD, *Elliptic problems in nonsmooth domains*, Pitman Advanced Publishing Program, 1985.

[6] T. NISHITANI, M. TAKAYAMA, A characteristic initial boundary value problem for a symmetric positive system, *Hokkaido Math. J.* **25** (1996), 167–182.

[7] T. NISHITANI, M. TAKAYAMA, *Regularity of solutions to characteristic boundary value problem for symmetric systems*, Geometrical Optics and Related Topics (F. Colombini, N. Lerner, eds.), Birkhäuser, 1997.

[8] J. RAUCH, Boundary value problems with nonuniformly characteristic boundary, *J. Math. Pures Appl.* **73** (1994), 347–353.

[9] S. SCHOCHET, The compressible Euler equations in a bounded domain: existence of solutions and the incompressible limit, *Comm. Math. Phys.* **104** (1986), 49–75.

[10] P. SECCHI, Well-posedness of characteristic symmetric hyperbolic systems, *Arch. Rat. Mech. Anal.* **134** (1996), 155–197.

[11] P. SECCHI, A symmetric positive system with nonuniformly characteristic boundary, *Diff. Int. Eq.* **11** (1998), 605–621.

[12] P. SECCHI, Inflow-outflow problems for inviscid compressible fluids, *Comm. Appl. Anal.* **2** (1998), 81–110.

[13] P. SECCHI, Full regularity of solutions to a nonuniformly characteristic boundary value problem for symmetric positive systems, *Adv. Math. Sci. Appl.* **10** (2000), 39–55.

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